Math 131- Spring 2017- Exam 2

- 6 True/False questions worth 2 points each.
- 12 multiple choice questions worth 4 points each.
- 4 hand graded questions worth 10 points each.

- No notes allowed. You may use one of the approved non-graphing calculators.
- True/False and Multiple Choice: Mark your answer on the answer card in pencil.
- Written: To receive full credit, write up a clear, complete solution, showing all steps.

7. Let $f(x) = x\cos(x)$. What is $f'(0)$?

(a) π

(b) -1

(c) 0

(d) 1

(e) none of the above

$f'(x) = \cos(x) + x \cdot (-\sin x)$

$f'(0) = 1 + 0$

to compensate

8. What is $\lim_{x \to 0} \frac{\sin(2x)\sin(3x)}{2x^2}$? $= \lim_{x \to 0} 3 \cdot \frac{\sin 2x}{2x} \cdot \frac{\sin(3x)}{3x} = 3 \cdot 1 \cdot 1 = 3$

(a) -3

(b) -2

(c) -1

(d) 0

(e) 1

(f) 2

(g) 3

(h) 6

(i) none of the above

11. Suppose $h(x) = sin(\pi f(x))$ and that $f(0) = 1$ and $f'(0)$=3. What is $h'(0)$?

(a) -4π
(b) -3π
(c) -2π
(d) $-\pi$
(e) 0
(f) π
(g) 2π
(h) 3π
(i) 6π
(j) none of the above

$\to h'(x) = \cos(\pi f(x)) \cdot \pi f'(x)$

$h'(0) = \cos(\pi f(0)) \cdot \pi f'(0) = \cos(\pi) \cdot \pi \cdot 3$

$= -1 \cdot \pi \cdot 3$

12. Let $f(x) = x^2 + e^{x-2}$. Then $P(2, 5)$ is a point on the graph. What is the equation for the tangent line at P?

(a) $y = x + 1$
(b) $y = x - 1$
(c) $y = 5$
(d) $y = 2x + e^{x-2}$
(e) $y = 5(x - 1)$
(f) none of the above

$f'(x) = 2x + e^{x-2}$ (Notice $\frac{d}{dx}(x-2) = 1$)

$f'(2) = 2 \cdot 2 + e^{2-2} = 4 + e^0 = 4 + 1 = 5$

$y - 5 = 5(x - 2)$

$y = 5x - 10 + 5 = 5x - 5$

$y = 5(x-1)$

15. Let $f(x) = \sin(x)$. Then the 101^{st} derivative $f^{(101)}(x)$ is

(a) $\sin(x)$

(b) $\cos(x)$

(c) $-\sin(x)$

(d) $-\cos(x)$

(e) none of the above

1st: $\cos x$ 5 9 ... 97 $\boxed{101}$
2nd: $-\sin x$ 6 10 ... 98
3rd: $-\cos x$ 7 11 ... 99
4th: $\sin x$ 8 12 16 20 24 100

16. Let $f(x) = e^{(x^2)}$. For what x does the graph of f have a horizontal tangent line?

(a) $x = -3$

(b) $x = -2$

(c) $x = -1$

(d) $x = 0$

(e) $x = 1$

(f) $x = 2$

(g) $x = 3$

(h) $x = 6$

(i) none of the above

$f'(x) = 2xe^{x^2} = 0 \Rightarrow 2x = 0$ or $e^{x^2} = 0$

$x = 0$ no solution

Written Problem. Clearly show all steps to receive full credit.

19. The position of a particle is given by $s = f(t) = t^3 - 6t^2 + 9t$ where t is in seconds and s is in meters.

(a) Find the velocity of the particle as a function of t.

$$V(t) = f'(t) = \boxed{3t^2 - 12t + 9}$$

(b) Find the acceleration of the particle as a function of t.

$$a(t) = f''(t) = \boxed{6t - 12}$$

(c) List the time intervals when the particle was moving forward.

Stopped when $V(t) = 0 \longrightarrow 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0 \Rightarrow t = 1, 3$

Possible Intervals $\boxed{(-\infty, 1)}$, $\cancel{(1, 3)}$, $\boxed{(3, \infty)}$

Check: $f'(0) = 9 > 0$ $\quad f'(2) < 0$ $\quad f'(4) > 0$

(d) Find the total distance (forward and backward) it moves in the first five seconds. (Note that this is not the same as the position at five seconds.)

$$\text{distance} = |f(1) - f(0)| + |f(3) - f(1)| + |f(5) - f(3)| = \boxed{28m}$$

Written Problem. Clearly show all steps to receive full credit.

21. For each function below, find $f'(x)$ using the rules of derivatives. Circle your final answer.

(a) $f(x) = 3x^2 - x + 2$ $\qquad f'(x) = \boxed{6x-1}$

(b) $f(x) = 10(x^3+9)^{24}$ $\qquad f'(x) = 240(x^3+9)^{23}\cdot(3x^2) = \boxed{720x^2(x^3+9)^{23}}$

(c) $f(x) = \ln(x^3+4x+12)$ $\qquad f'(x) = \boxed{\dfrac{3x^2+4}{x^3+4x+12}}$

(d) $f(x) = \frac{e^x+\sec(x)}{2^x}$

$$f'(x) = \frac{(e^x+\sec x\tan x)2^x - \ln 2\cdot 2^x(e^x+\sec x)}{2^{2x}}$$

$$= \boxed{\frac{e^x+\sec x\tan x - \ln 2\, e^x \mp \ln 2\sec x}{2^x}}$$

(e) $f(x) = \tan^{-1}(2x)$ $\qquad f'(x) = \dfrac{1}{1+(2x)^2}\cdot 2 = \boxed{\dfrac{2}{1+4x^2}}$

use the rule for $\tan^{-1}x$,
or derive with implicit differentiation.